Hosotani mechanism redux

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Kashiwa, TM, [arXiv:1302.2196].
Kouno, TM, Kashiwa, Makiyama, Sasaki, Yahiro, in preparation.

03/21/2013@Brain workshop

Dynamical Gauge Symmetry Breaking

→ *Key to Understanding of Modern Physics Frontier*

GUT ?

↓

DGBS at GUT scale

$SU(3)_C \times SU(2)_L \times U(1)_Y$

↓

DGBS at EW scale

$\boxed{SU(3)_C} \times \boxed{U(1)_{EM}}$

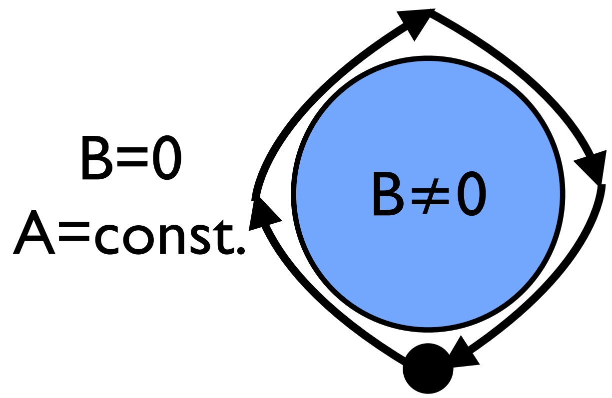
CSC SC

DGBS at extreme cond.

CW mechanism, Technicolor, ***Gauge-Higgs*****.....**

◆AB effect & Hosotani mechanism

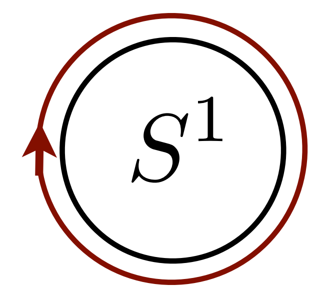
Wilson-loop phase : $\delta\phi = \frac{e}{\hbar}\int_C A dx \longleftrightarrow e^{i\delta\phi} \sim W = P\exp(ig\int_C A dx)$



Constant gauge field can affect physics thru
Gauge-invariant phases

⇨ **Extension to *SU(N)* on** $R^d \times S^1$

Wilson loop in S^1 : $W = P\exp\left\{ig\int_C dy A_y\right\}$



1. *N* eigenvalues : *N* wilson-loop phases $[e^{2\pi i q_1}, e^{2\pi i q_2}, \cdots, e^{2\pi i q_N}]$
2. Gauge-invariant as long as keeping B.C.
3. **Can affect physics**

$q_i \neq q_j$ ⇨ **spontaneous gauge symmetry breaking**

e.g.) $q_1 = q_2 = q_3 \neq q_4 = q_5$, SU(5)→SU(3)×SU(2)×U(1)

1. Gauge-Higgs unification in (4+1)D

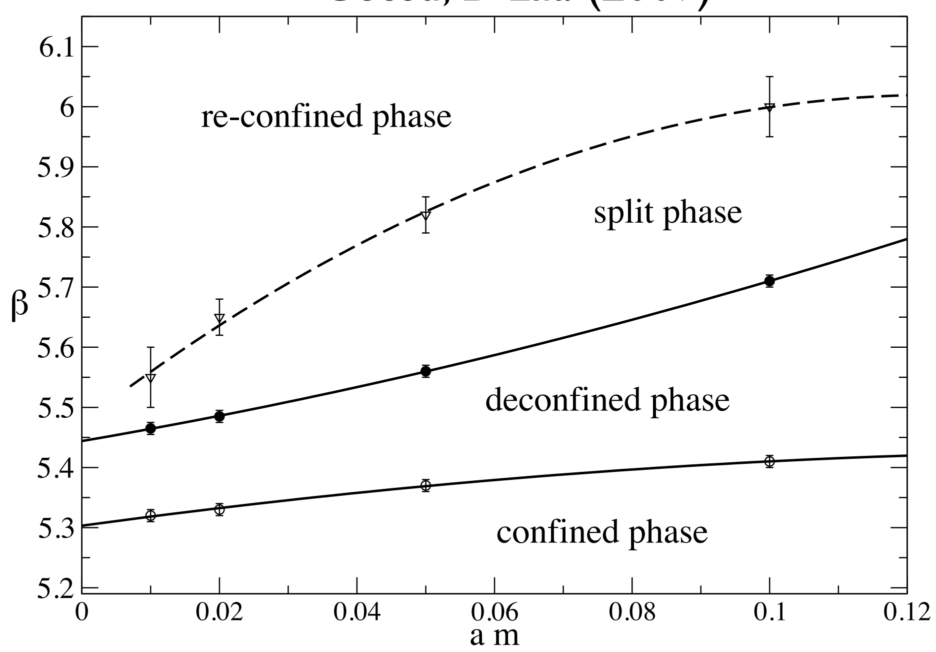
$q_i \neq q_j$ → Higgs as gauge field in S^1 ($m_H \sim O(g/L)$)

- KK spectrum $m_n^2 = \frac{1}{L^2}(n + q_i - q_j)^2$ → massive gauge boson
- Determined dynamically depending on matter (Adj. needed)

2. Finite-T QCD with PBC adj. in (3+1)D

Myers-Ogilvie (2007)
Cossu-D'Elia (2009)

Cossu, D'Elia (2009)



- Rich phase structure found !
- Should be understood from Hosotani mechanism.

Purpose

1. Understand phase structure of QCD-like theories in terms of GSB.

2. Obtain useful information for GH models.

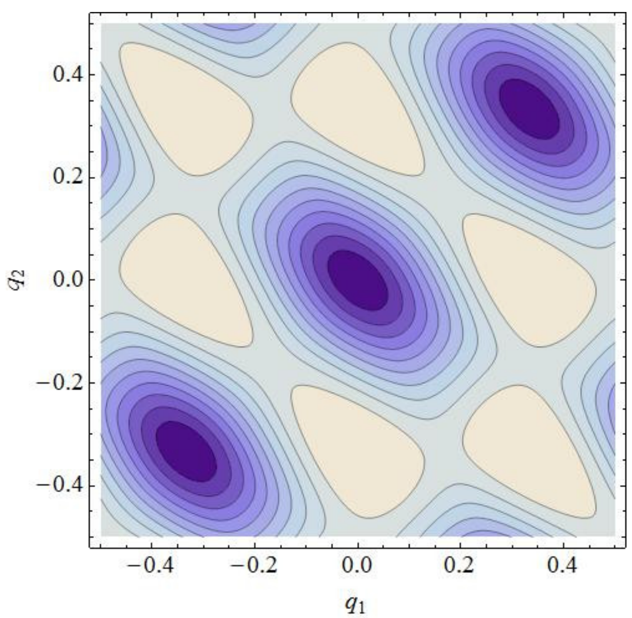
3. Seek new setups leading to GSB.

Study on GH from QCD phase diagram

Tools
- One-loop effective potential
- Polyakov-loop-extended NJL

◆Gauge-unbroken cases

- Contour plot of pure SU(3) potential on $R^d \times S^1$



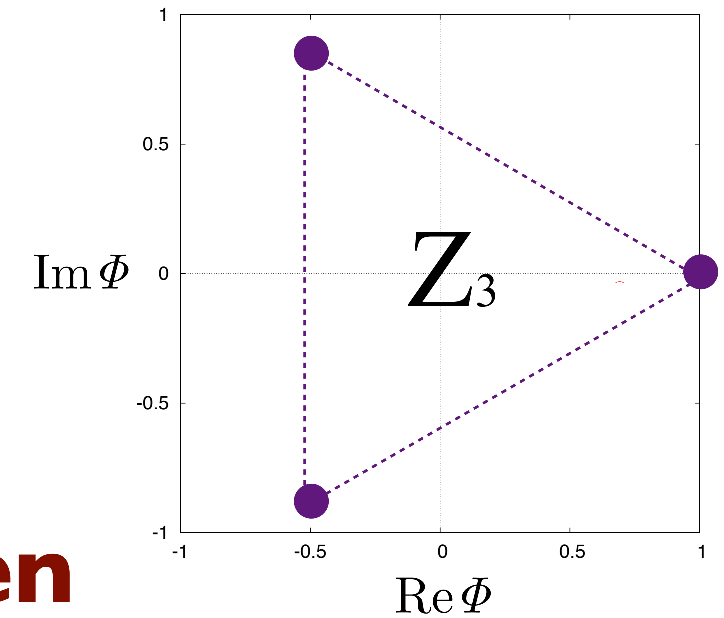
$$(q_1, q_2, q_3) = (1/3, 1/3, 1/3)$$

$$(q_1, q_2, q_3) = (0, 0, 0)$$

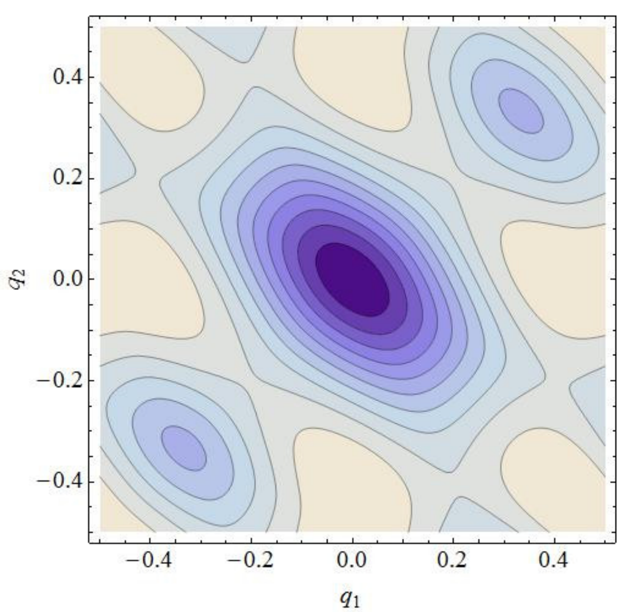
$$(q_1, q_2, q_3) = (-1/3, -1/3, -1/3)$$

$$\boxed{q_1 = q_2 = q_3}$$

SU(3) unbroken

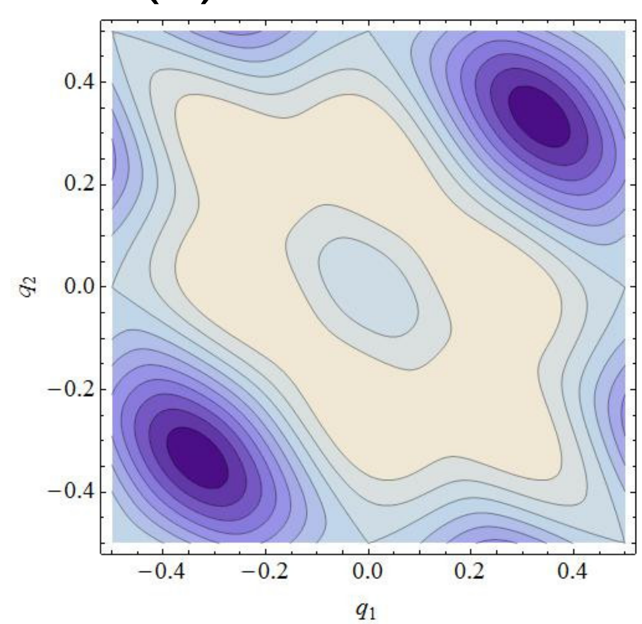


(1) aPBC fund.



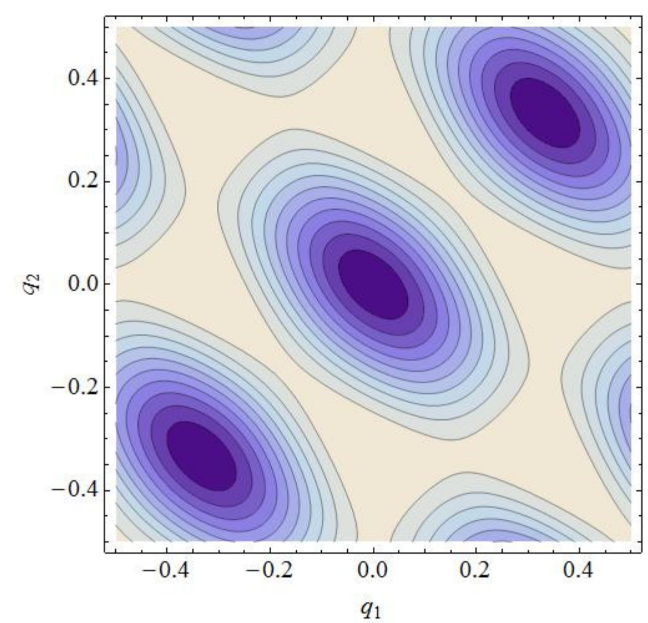
$\cancel{Z_3}$

(2) PBC fund.



$\cancel{Z_3}$

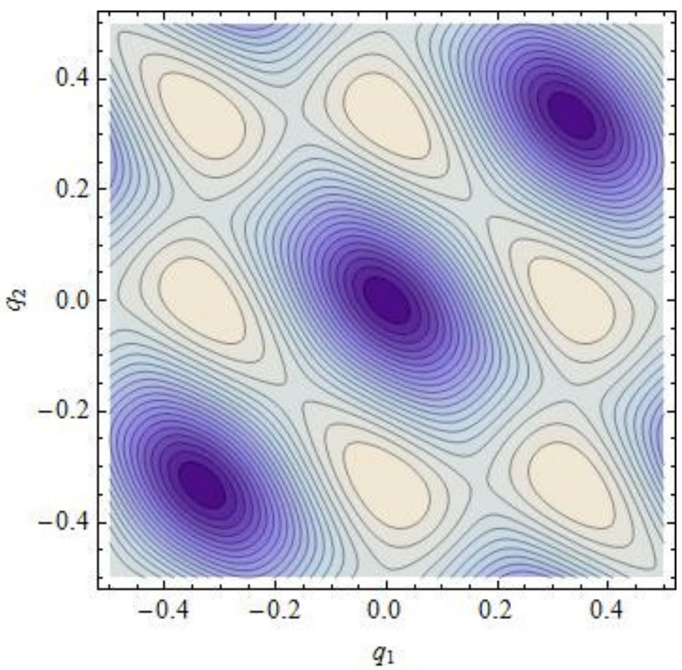
(3) aPBC adj.



Z_3

SU(3) is intact in all these cases.

◆Gauge-broken case : PBC adj.

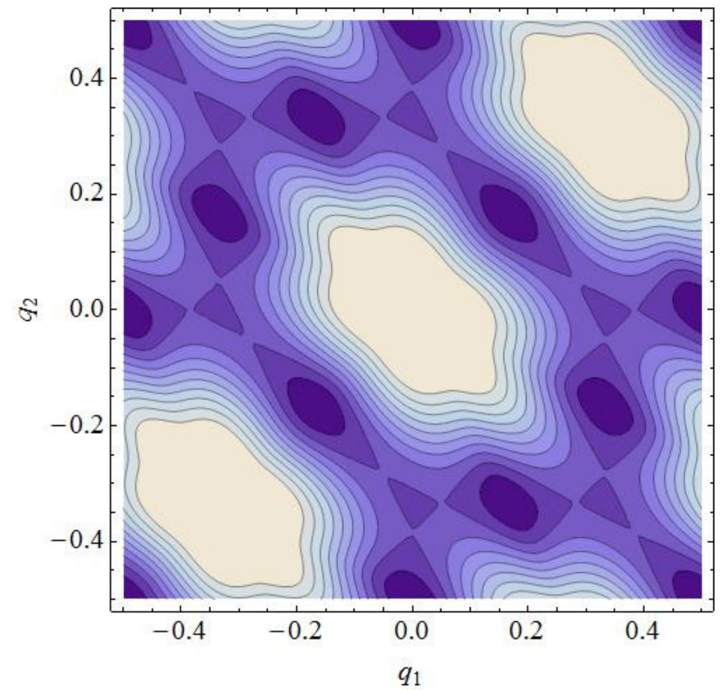


large m

$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$ $(0,0,0)$ $(-\frac{1}{3},-\frac{1}{3},-\frac{1}{3})$

$$q_1 = q_2 = q_3$$

SU(3)

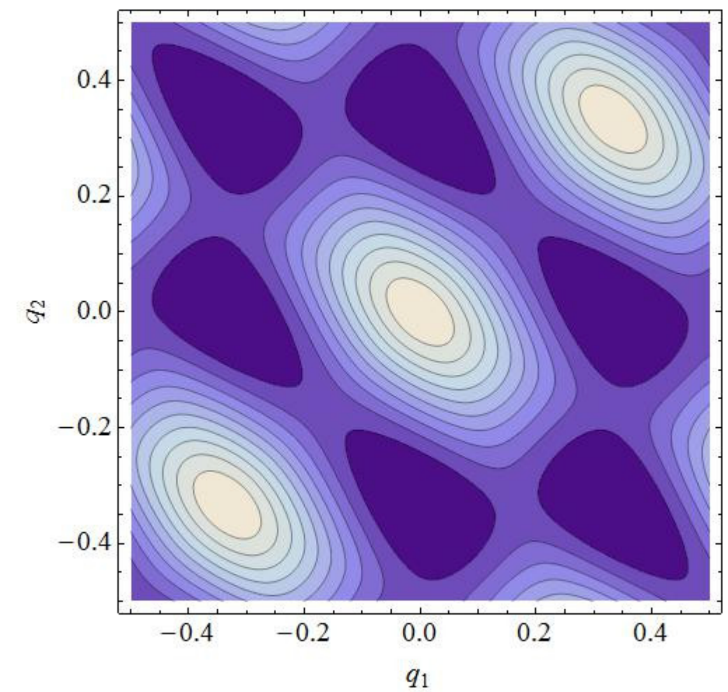


medium m

$(\frac{1}{2},\frac{1}{2},0)$ $(\frac{1}{6},\frac{1}{6},-\frac{1}{3})$ $(-\frac{1}{6},-\frac{1}{6},\frac{1}{3})$

$$q_1 = q_2 \neq q_3$$

SU(2)×U(1)



small m

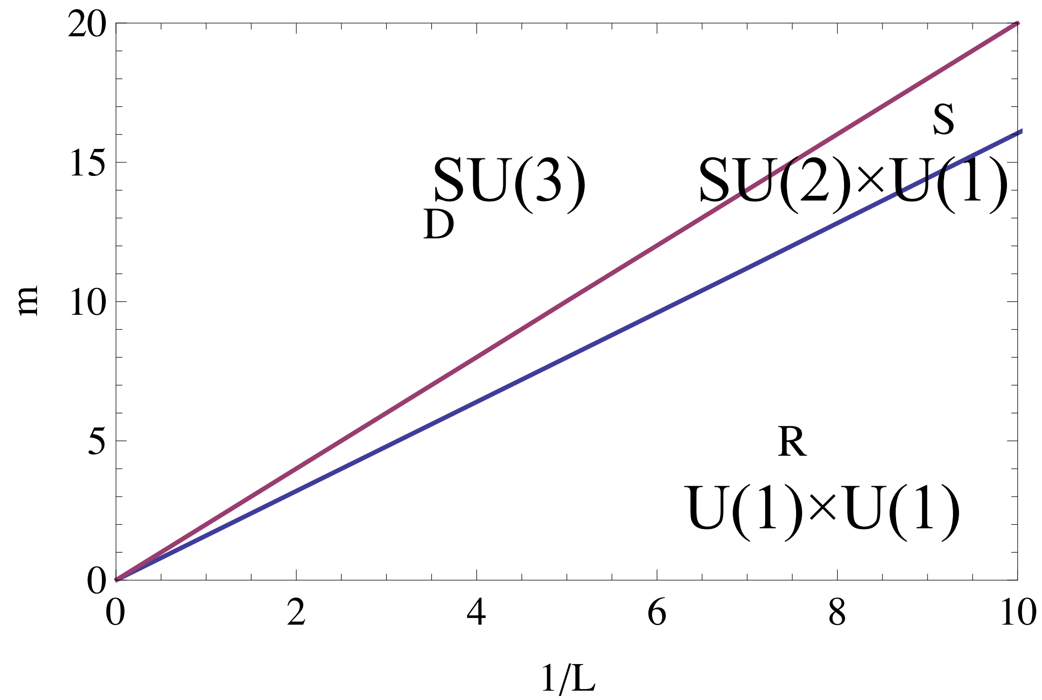
$(\frac{1}{3},-\frac{1}{3},0)$

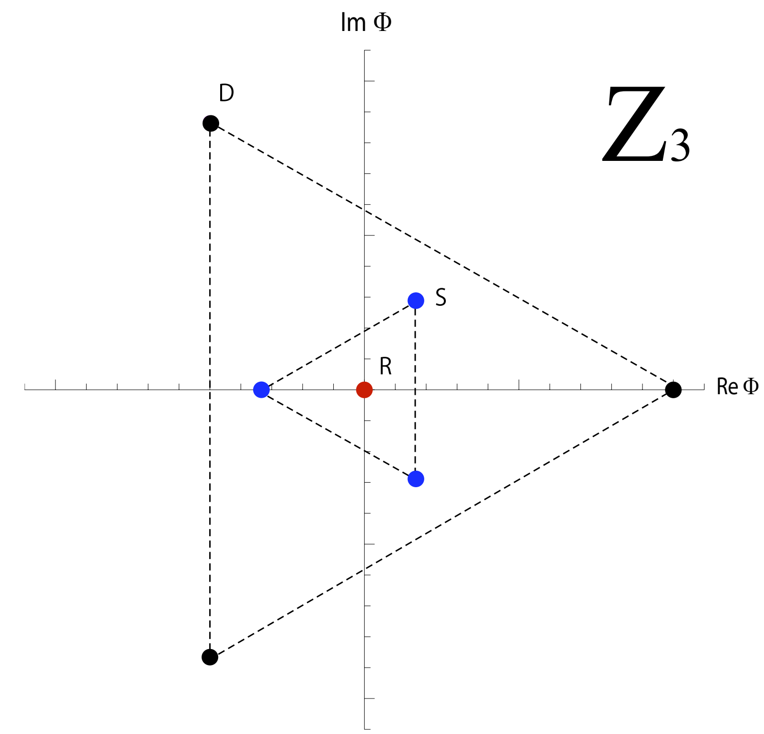
$$q_1 \neq q_2 \neq q_3$$

U(1)×U(1)

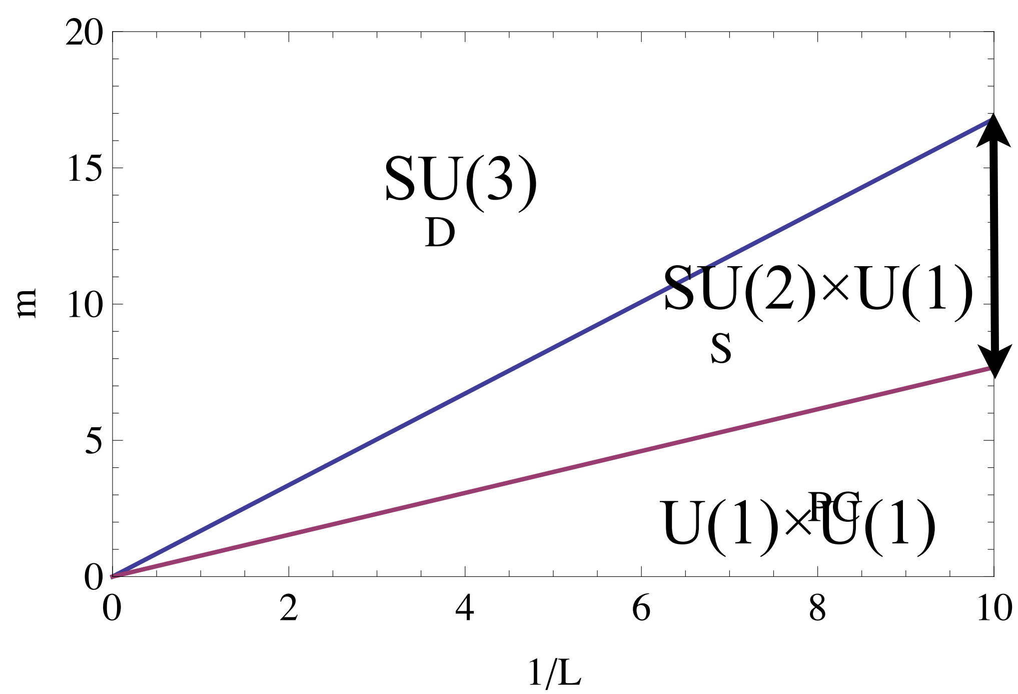
SU(3) is broken depending on mass & compactification scale

◆Phase diagram & Polyakov-loop

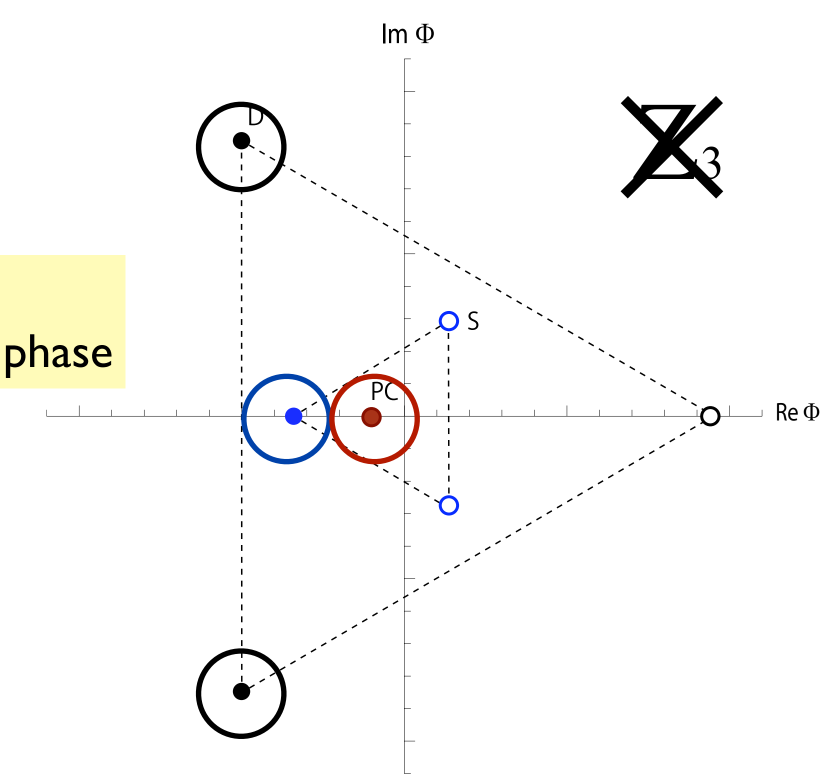




◆SU(3) with adj. & fund. with PBC

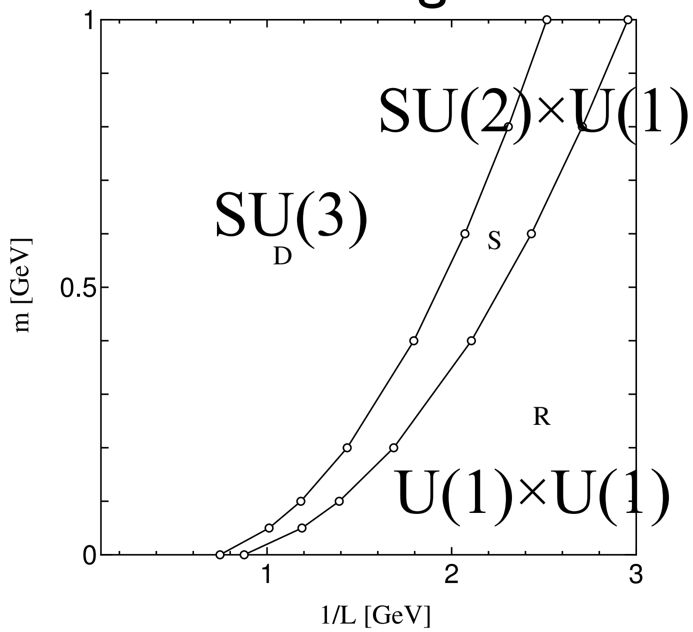


Wider
SU(2)×U(1) phase

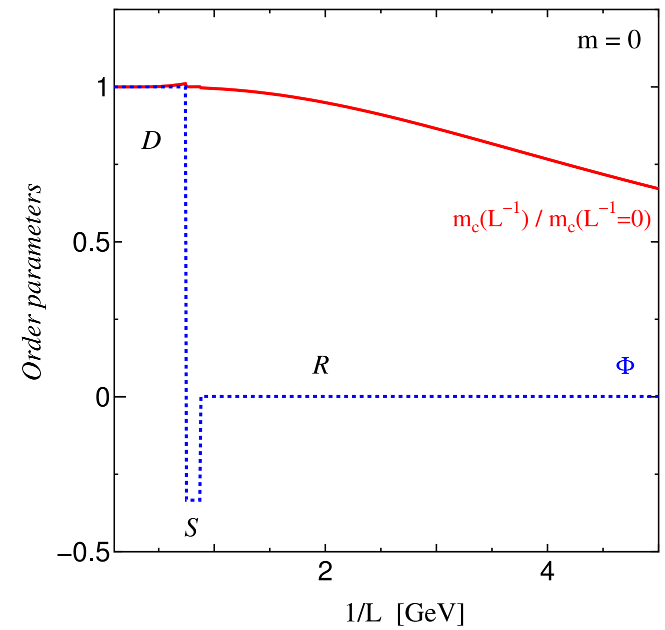


◆Chiral properties ← PNJL model

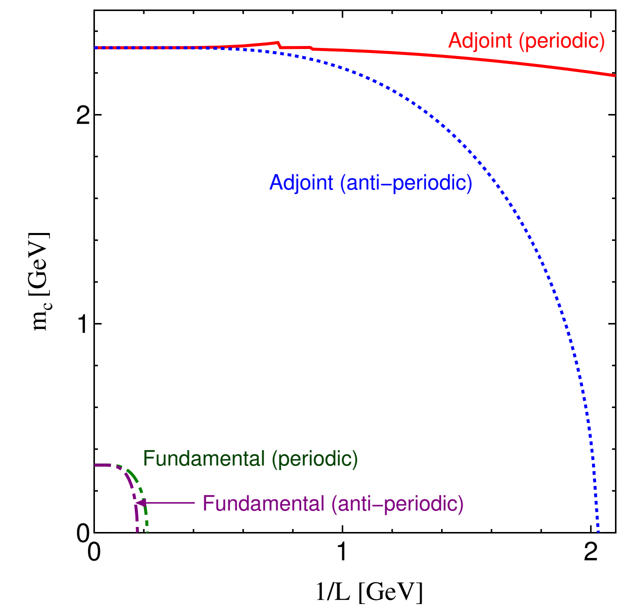
Phase diagram



chiral condensate



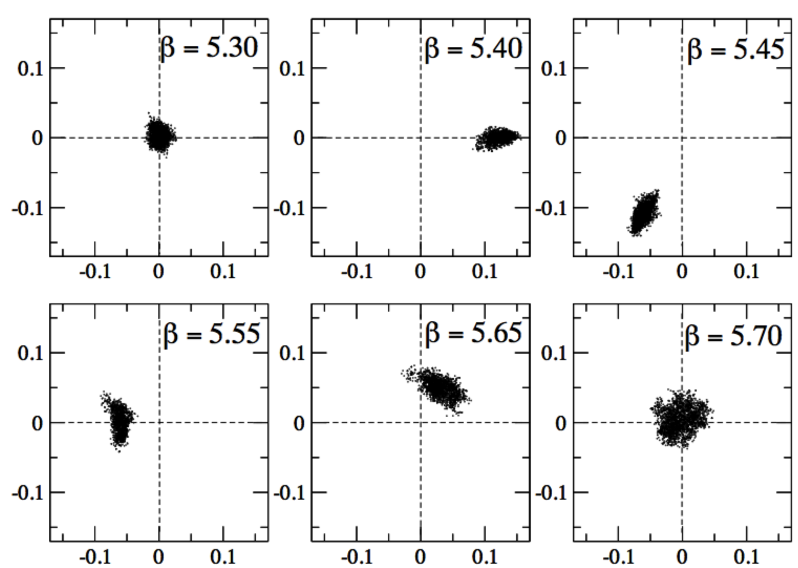
comparison to other cases



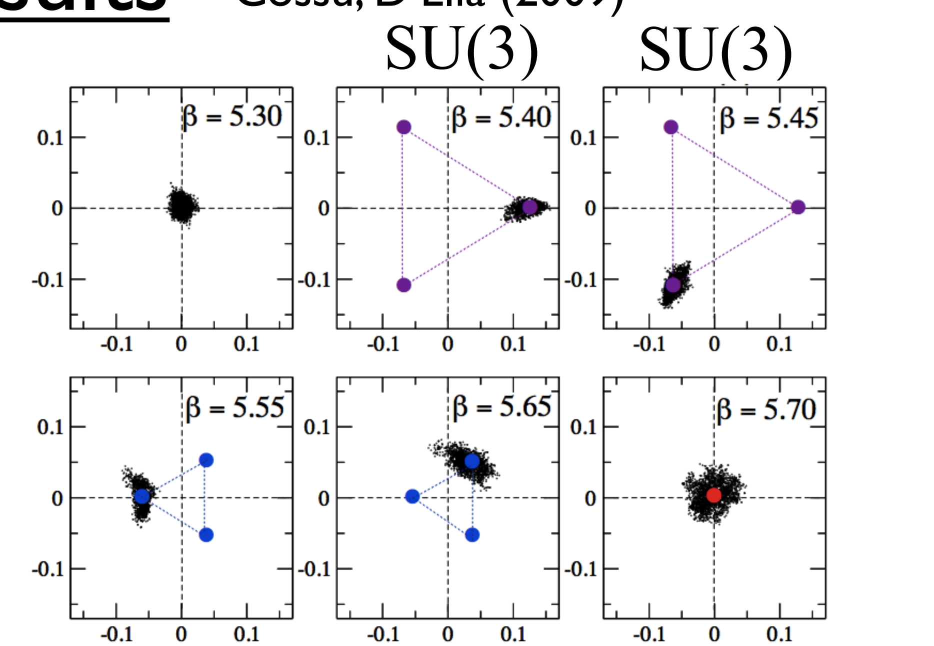
Chiral symmetry is slowly restored in GSB phase of PBC-Adj.

◆Re-interpretation of Lattice results

Cossu, D'Elia (2009)



Polyakov-loop plot



◆ Fund. quarks can break gauge symmetry?

→ Flavor Twisted Boundary Conditions

$$(q_1, q_2, q_3)_{x,y+L} = (q_1, e^{2\pi i/3} q_2, e^{4\pi i/3} q_3)_{x,y}$$

cf.) Flavored Im. chemical potential

Z_3 center is preserved by use of Z_3 of flavor SU(3).

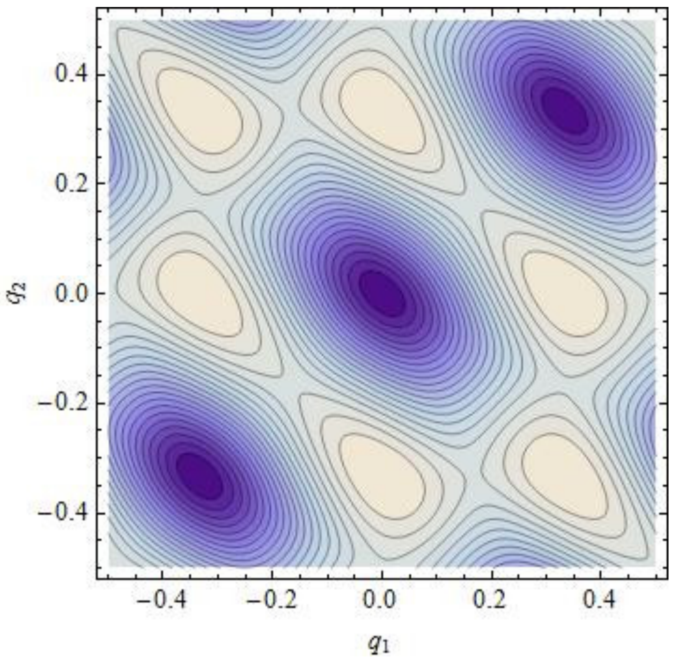
- One-loop effective potential for FTBC

$$\mathcal{V}_f^{FT} = +\frac{4}{L^4\pi^2} \sum_i^3 \sum_f^3 \sum_{n=1}^{\infty} \frac{\cos[2\pi n q_{if}]}{n^4} \qquad q_{if} = q_i + (f-1)/3$$

similar to PBC adj.

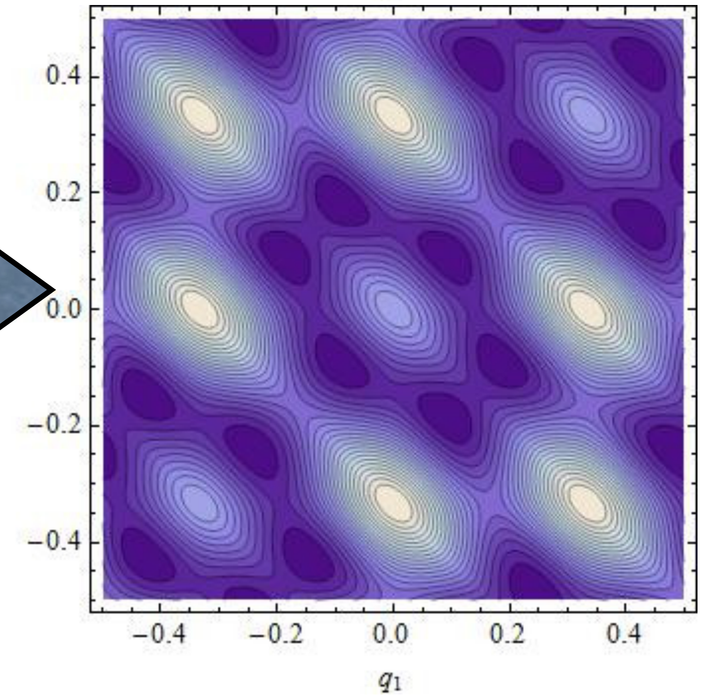
cf.) $\mathcal{V}_a = +\frac{4}{L^4\pi^2} \sum_{i,j=1}^{3} \sum_{n=1}^{\infty} \left(1 - \frac{1}{3}\delta_{ij}\right) \frac{\cos[2\pi n q_{ij}]}{n^4}$

GSB in FTBC



large m

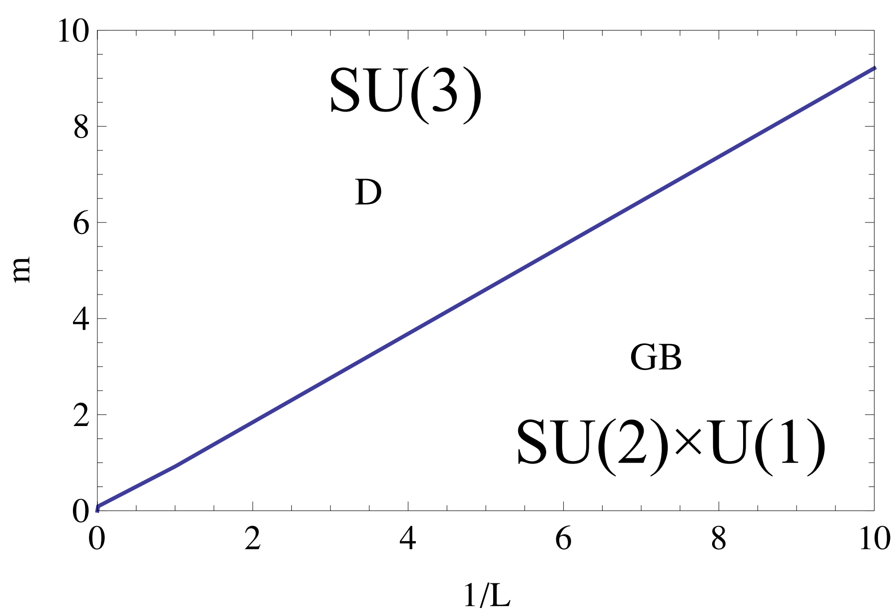
SU(3)

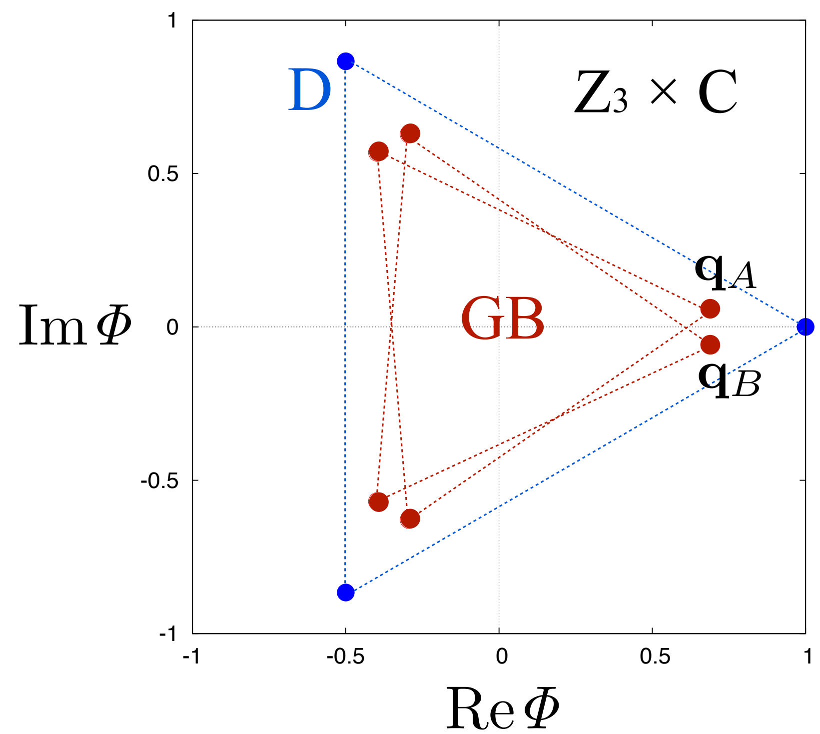


small m

SU(2)×U(1)

$q_1 = q_2 \neq q_3$





Ideal GSB of SU(3) to SU(2)×U(1)

Summary

1. Rich phase structure with SGSB in gauge theory on compactified space with PBC.

2. Fundamental flavors with PBC works to enhance SU(2)×U(1) phase.

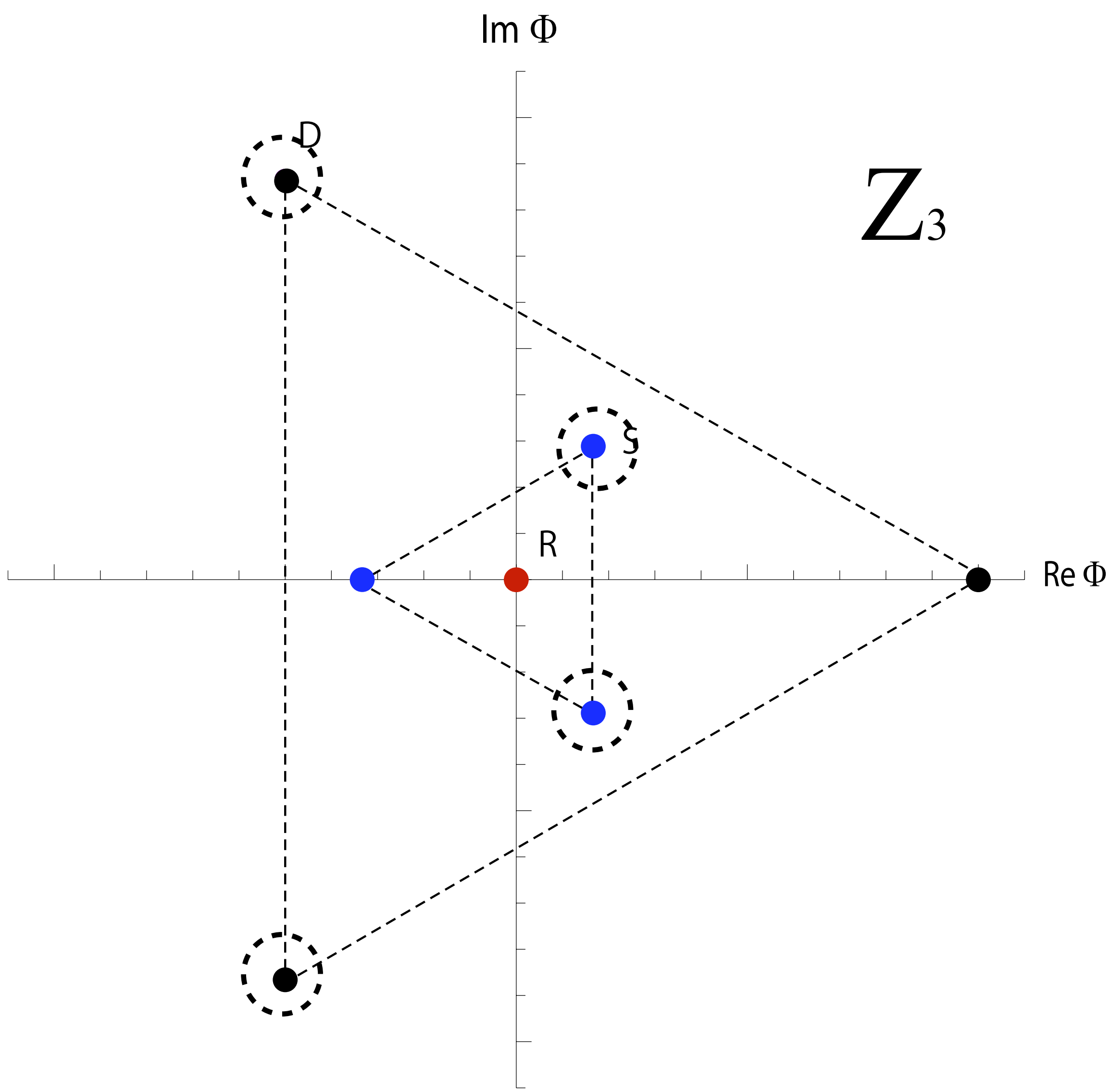
3. Fund. fermions with FTBC also leads to SU(2)×U(1) SGSB.

4. Specific chiral properties.

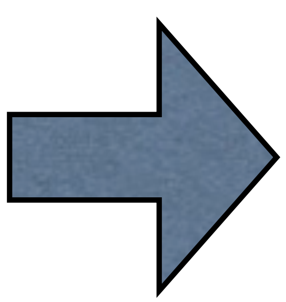
Future works

- Further lattice study 4D to check our results.
- Lattice study for 5D as cutoff theory.
- Application of FTBC to BSM, QCD....

C breaking problem



ImΦ ⇆ -ImΦ breaking



Charge conjugation breaking ?

Elitzur's theorem

$\langle A \rangle = 0$ on the lattice

$\langle P \rangle \neq 0$ on the lattice

- DGSB by Hosotani mechanism is topological phenomenon.
- Can be indirectly observed from Gauge-invariant quantity.

1. Gauge breaking deformation w/ parameter
2. Extrapolation of the parameter to zero

SU(3) adj. with non-perturbative deformation

$$\mathcal{V}_g^{\rm np} = -\frac{2}{L^4\pi^2}\sum_{i,j=1}^{N}\sum_{n=1}^{\infty}\Big(1-\frac{1}{N}\delta_{ij}\Big)\frac{\cos(2n\pi q^{ij})}{n^4} + \frac{M^2}{2\pi^2 L^2}\sum_{i,j=1}^{N}\sum_{n=1}^{\infty}\Big(1-\frac{1}{N}\delta_{ij}\Big)\frac{\cos(2n\pi q^{ij})}{n^2}$$